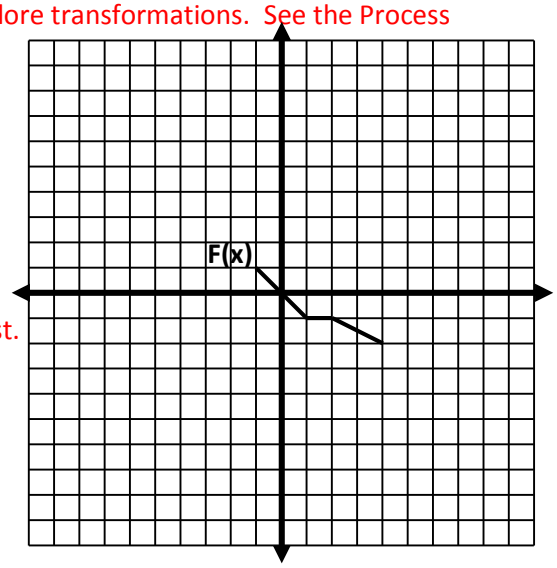
Transformations with Fred Functions – Day 1 KEY/TEACHER NOTES

To the right is a graph of a **"Fred" function**. We can use Fred functions to explore transformations in the coordinate plane. Fred is any generic function that we can use to explore transformations. See the Process Geometry book for more information on Fred if needed or wanted.



I. Let's review briefly.

1. a. Explain what a function is in your own words.
   A function is a relation in which every element of the domain (input) maps to exactly one element of the range (output).
   Since a graph is shown, students may mention the vertical line test.
   b. Using the graph, how do we know that Fred is a function?
   It passes the vertical line test.
   (Every x-value has exactly one y-value.)
2. a. Explain what we mean by the term domain.

   the set of all inputs (x-values) of a function or relation

   b. Using the graph, what is the domain of Fred?

   $\{x \mid -1 \leq x \leq 4\}$ …. Teachers may or may not want to make students use the formal set notation.

3. a. Explain what we mean by the term range.

   the set of all inputs (x-values) of a function or relation

   b. Using the graph, what is the range of Fred?

   $\{y \mid -2 \leq y \leq 1\}$ …. Teachers may or may not want to make students use the formal set notation.

4. Let's explore the points on Fred.
   a. How many points lie on Fred? Infinite number* Can you list them all? no

   b. What are the key points that would help us graph Fred?

   (–1, 1), (1, –1), (2, –1), (4, –2)

   * Students may say that there are 4 points on Fred. Be ready to discuss this.

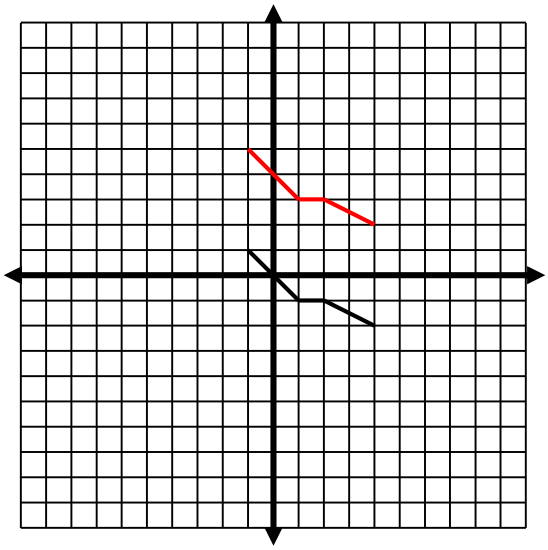
   We are going to call these key points **"characteristic" points**. It is important when graphing a function that you are able to identify these characteristic points.

   c. Use the graph of graph to evaluate the following.
   F(1) = __–1___ F(–1) = __1___ F(_4___) = –2 F(5) = __undefined____

II. Remember that F(x) is another name for the y-values.

Therefore the equation of Fred is **y = F(x)**.

x	F(x)
–1	1
1	–1
2	–1
4	–2



1. Why did we choose those x-values to put in the table?
   These are the characteristic points of Fred and are key to drawing the various parts of the graph.

   Now let's try graphing Freddie Jr.: **y = F(x) + 4**. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.

y = F(x) + 4

x	y
–1	5
1	3
2	3
4	2

2. What type of transformation maps Fred, F(x), to Freddie Jr., F(x) + 4? (Be specific.)
   Translation up 4 units
   (Remember, we are trying to make a connection to Unit 1's transformation study.)

3. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   There was no change in x-values.

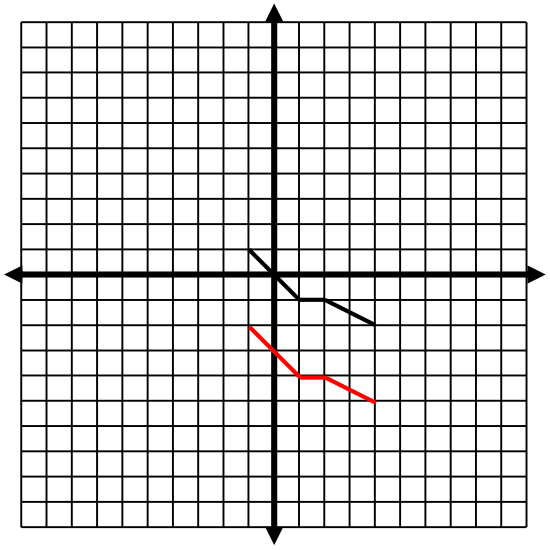
4. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   Each of the y-values increased by 4.

5. In y = F(x) + 4, how did the "+4" affect the graph of Fred? Did it affect the domain or the range?
   It translated Fred up 4 units. It affected the range.

III. Suppose Freddie Jr's equation is: **y = F(x) – 3**. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.

y = F(x) – 3

x	y
–1	–2
1	–4
2	–4
4	–5



1. What type of transformation maps Fred, F(x), to Freddie Jr., F(x) – 3? Be specific.
   Translation down 3 units

2. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   There was no change in x-values.

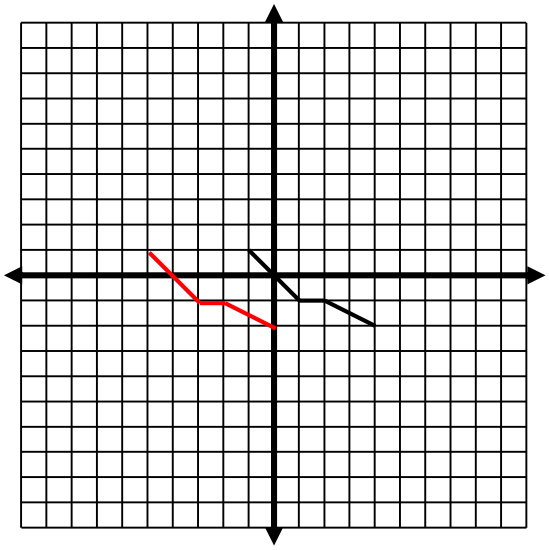
3. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   Each y-value decreased by 3.

4. In y = F(x) – 3, how did the "– 3" affect the graph of Fred? Did it affect the domain or the range?
   It translated Fred down 3 units.

IV. Checkpoint: Using the understanding you have gained so far, describe the affect to Fred for the following functions.

Equation	Effect to Fred's graph
Example: y=F(x) + 18	Translate up 18 units
1. y = F(x) – 100	Translate down 100 units
2. y = F(x) + 73	Translate up 73 units
3. y = F(x) + 32	Translate up 32 units
4. y = F(x) – 521	Translate down 521 units

V. Suppose Freddie Jr's equation is: **y = F(x + 4)**.



1. Complete the table.

x	x + 4	y
–5	–1	1
– 3	1	–1
–2	2	–1
0	4	–2

*(**Hint**: Since, x + 4 = –1, subtract 4 from both sides of the equation, and x = –5. Use a similar method to find the missing x values.)*

2. On the coordinate plane above, graph the 4 ordered pairs (x, y). The first point is (–5, 1).
   You may want to suggest or require that students list the 4 points before trying to graph them so they do not mistakenly use that middle column from the chart instead of the x and y columns.
3. What type of transformation maps Fred, F(x), to Freddie Jr., F(x + 4)? (Be specific.)
   Translation left 4 units

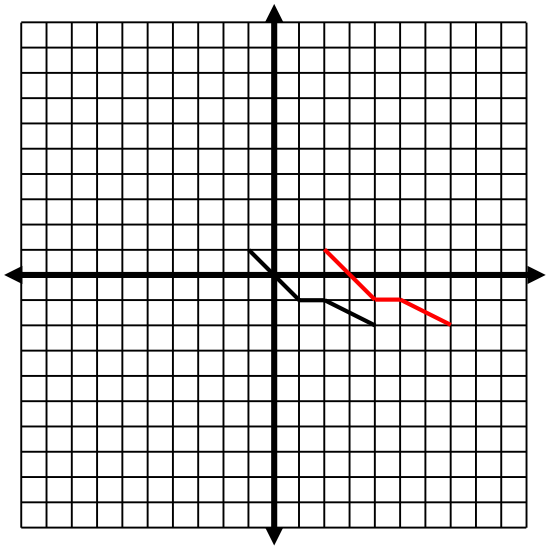
4. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   Each x-value decreased by 4.

5. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   There was no change in y-values.

6. In y = F(x + 4), how did the "+4" affect the graph of Fred?

It translated Fred left 4 units. (Note with students that the effect of the +4 this time yields a horizontal movement since it is the x-values that are being increased by 4.)

VI. Suppose Freddie Jr's equation is: **$y = F(x - 3)$**. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.



1. Complete the table.

$y = F(x - 3)$

x	x – 3	y
2	–1	1
4	1	–1
5	2	–1
7	4	–2

Remind students that the y-values for those input values in the middle column are the same as the original graph of Fred. They may still need help in figuring out that they need to add 3 to the middle column to get the x-values in the left column.

2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st point should be (2, 1).*]
   Again, listing them first may help in graphing them correctly.
3. What type of transformation maps Fred, F(x), to Freddie Jr., F(x – 3)? (Be specific.)
   Translation right 3 units

4. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   It increased them by 3.

5. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   There was no change to the y-values.

6. In $y = F(x - 3)$, how did the " –3" affect the graph of Fred?
   It translated Fred right 3 units.

   [If students focus on how we got our new x-values (by adding 3 to both sides of the equation), the reason the graph shifts right instead of left will be more logical to them.]

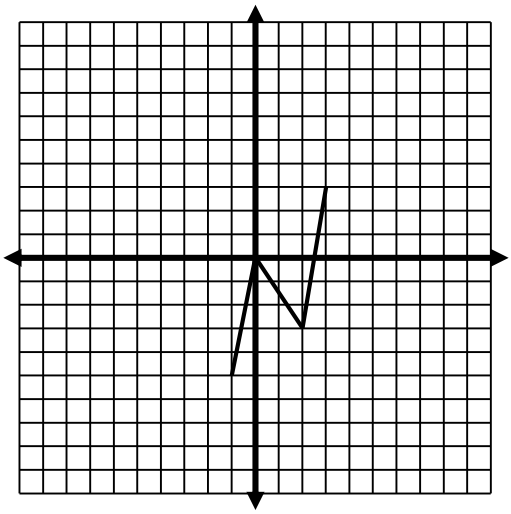
VII. Checkpoint: Using the understanding you have gained so far, describe the effect to Fred for the following functions.

Equation	Effect to Fred's graph
Example: y=F(x + 18)	Translate left 18 units
1. y = F(x – 10)	Translate right 10 units
2. y = F(x) + 7	Translate up 7 units
3. y = F(x + 48)	Translate left 48 units
4. y = F(x) – 22	Translate down 22 units
5. y = F(x + 30) + 18	Translate left 30 units and up 18 units

VIII. Checkpoint: Using the understanding you have gained so far, write the equation that would have the following effect on Fred's graph.

Equation		Effect to Fred's graph
Example:	y=F(x + 8)	Translate left 8 units
1.	y=F(x) + 29	Translate up 29 units
2.	y=F(x – 7)	Translate right 7
3.	y=F(x + 45)	Translate left 45
4.	y=F(x + 5) + 14	Translate left 5 and up 14
5.	y=F(x – 6) – 2 *	Translate down 2 and right 6

IX. Now let's look at a new function.
Its notation is **H(x)**, and we will call it **Harry**.
Use Harry to demonstrate what you have learned so far about the transformations of functions.



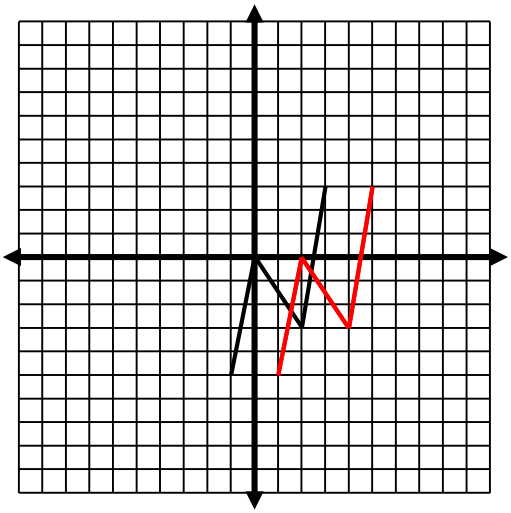
1. What are Harry's characteristic points?

___(–5, –1), (0, 0), (2, –3), (3, 3)_____

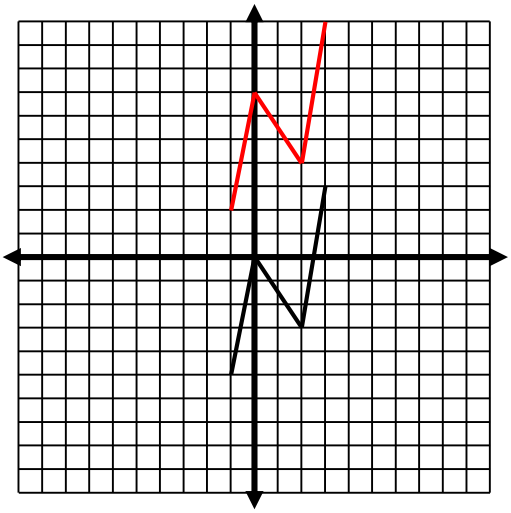
2. Describe the effect on Harry's graph for each of the following.
   a. H(x – 2) ___Translate right 2 units_____
   b. H(x) + 7 ___Translate up 7 units_____
   c. H(x+2) – 3 ___Translate left 2 units and down 3 units_____

3. Use your answers to questions 1 and 2 to help you sketch each graph *without using a table*.

a. y = H(x – 2)



b. y = H(x) + 7



c. y = H(x+2) – 3