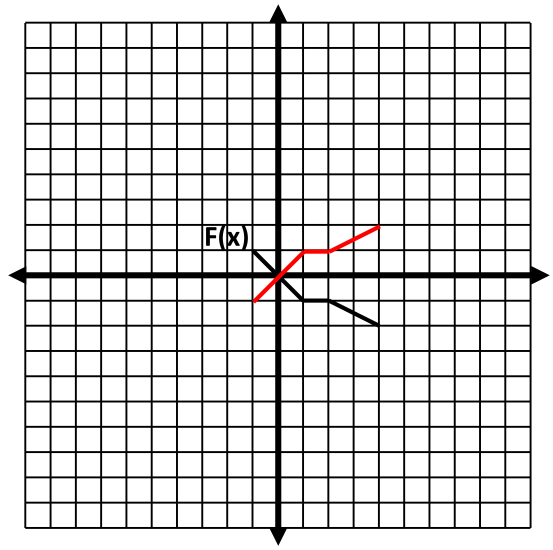
Transformations with Fred – Day 2 KEY/TEACHER NOTES

Today we will revisit Fred, our "parent" function, and investigate transformations other than translations.

Recall that the equation for Fred is **y = F(x)**.

Complete the chart with Fred's characteristic points.

x	F(x)
–1	1
1	–1
2	–1
4	–2



I. Let's suppose that Freddie Jr. is **y = – F(x)**

1. Complete the table.

y = – F(x)

x	F(x)	y
–1	1	–1
1	–1	1
2	–1	1
4	–2	2

Be sure students understand that the y-value is simply the opposite of F(x).

2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st point should be (–1, –1).*]
   These should be: (–1, –1), (1, 1), (2, 1), (4, 2)
3. What type of transformation maps Fred, F(x), to Freddie Jr., –F(x)? (Be specific.)
   Reflection in the x-axis
4. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   There was no change to the x-values.
5. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   The y-values changed to their opposites.
6. In y = – F(x), how did the negative coefficient of "F(x)" affect the graph of Fred? How does this relate to our study of transformations earlier this semester?
   It reflected Fred in the x-axis. To reflect in the x-axis, you keep the x-value and take the opposite of the y-value. The negative coefficient takes the opposite of the original F(x) , or y, value, but it does not affect the x-value. [Note: Be sure students understand that by negating F(x), (the y-values), the change will result in a vertical movement. Therefore, the reflection is over the x-axis.]

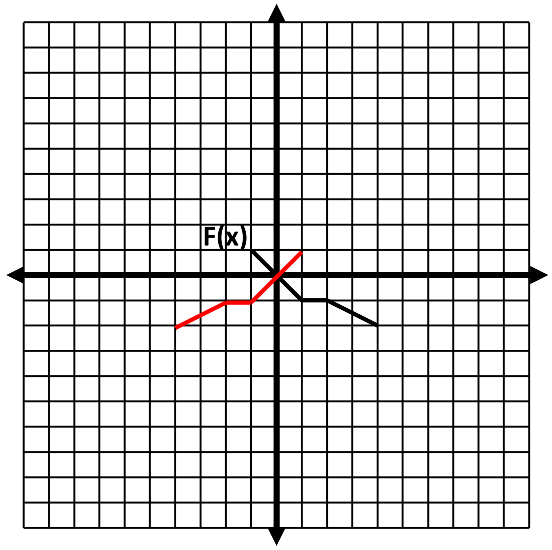
II. Now let's suppose that Freddie Jr. is **y = F(–x)**

1. Complete the table.

y = F(–x)

x	**–x**	**y**
1	–1	1
–1	1	–1
–2	2	–1
–4	4	–2

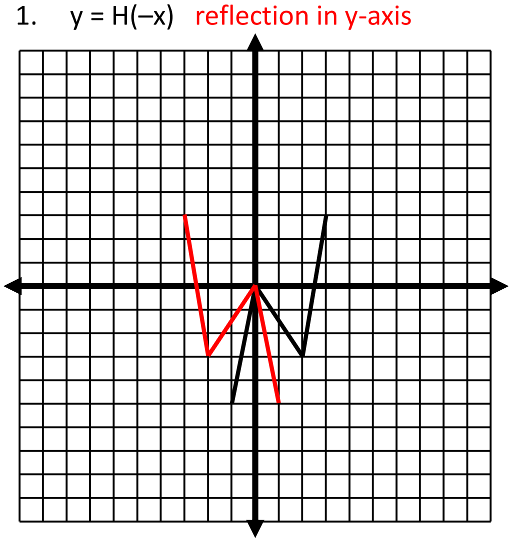
After Day 1's lesson, students may understand why the x-values from the characteristic points of Fred must be used for the "–x" column. However, you will want to check to be sure.



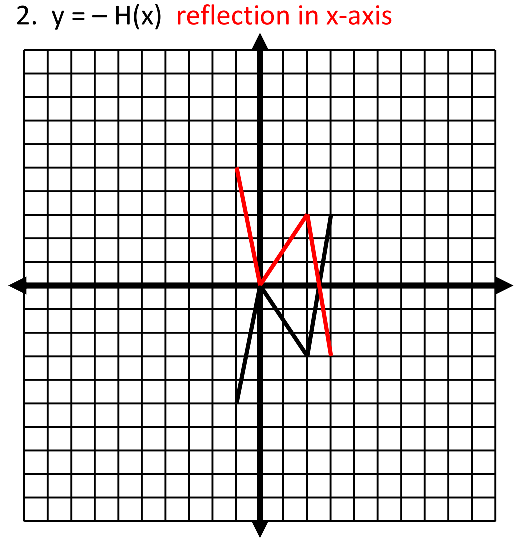
2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st point should be (1, 1).*]
   These should be: (1, 1), (–1, –1), (–2, –1), (–4, –2)
3. What type of transformation maps Fred, F(x), to Freddie Jr., F(–x)? (Be specific.)
   Reflection in the y-axis
4. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   The x-values changed to their opposites.
5. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   There was no change in the y-values.
6. In y = F(–x), how did the negative coefficient of "x" affect the graph of Fred? How does this relate to our study of transformations earlier this semester? It reflected Fred in the y-axis. To reflect in the y-axis, you keep the y-value and take the opposite of the x-value. The negative coefficient of x takes the opposite of the original x-value but does not affect the y-value.

III. **Checkpoint: Harry is H(x) and is shown on each grid. Use Harry's characteristic points to graph Harry's children without making a table.**

1. y = H(–x) reflection in y-axis



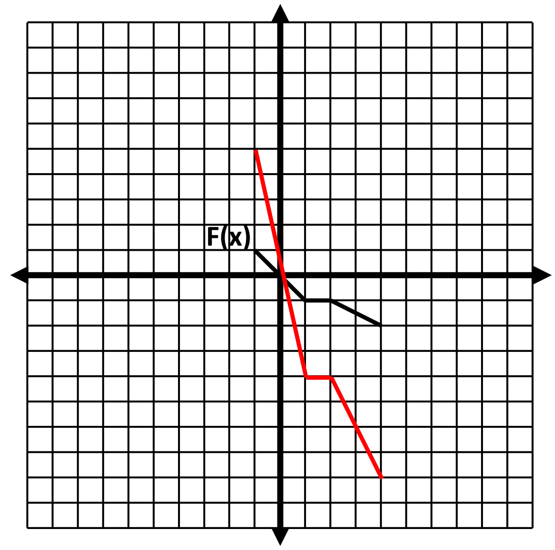
2. y = – H(x) reflection in x-axis



IV. Now let's return to Fred, whose equation is **y = F(x)**.

Complete the chart with Fred's characteristic points.

x	F(x)
–1	1
1	–1
2	–1
4	–2



Let's suppose that Freddie Jr. is **y = 4 F(x)**

1. Complete the table.

y = 4 F(x)

x	F(x)	y
–1	1	4
1	–1	– 4
2	–1	– 4
4	–2	– 8

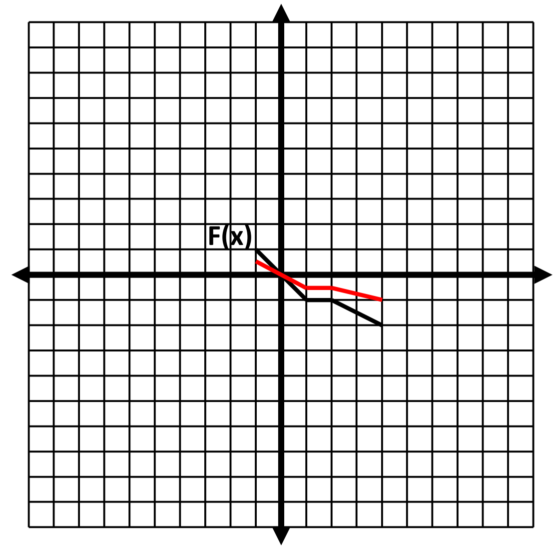
2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st one should be (–1, 4).*]
   These should be: (–1, 4), (1, – 4), (2, – 4), (4, – 8)
3. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   There was no change to the x-values.
4. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   Freddie Jr's y-values are 4 times the y-values of Fred.
5. In y = 4 F(x), the coefficient of "F(x)" is 4. How did that affect the graph of Fred? Is this one of the transformations we studied? If so, which one? If not, explain. The graph stretched vertically by a factor of 4.** It did not stretch horizontally. Each point is 4 times as far from the x-axis. It is not one of the transformations we studied. It did not preserve size, so it is neither a translation, reflection, nor rotation. It did not preserve shape, so it is not a dilation. (The x-values did not quadruple when the y-values did, so it is not a dilation with scale factor 4.)
   **** Students will often want to use the terms "skinnier/fatter" or "wider/narrower" with these type of questions. While the graphs do look that way, the effect on the graph by the coefficient of F(x) is actually changing the y-values for a given x, so the actual effect is that the graph changes vertically (stretches or compresses). If you can help lead students to the better terms of "stretches vertically" or "compresses vertically," you will help them in their understanding of how this coefficient affects the graph, which will help them when they progress to later units (including trig). [If you want to keep terminology simple for now, encourage "taller/shorter" or "steeper /less steep" that informally indicates a vertical change and relates back to their knowledge of slopes in linear equations.]**

V. Now let's suppose that Freddie Jr. is **y = ½ F(x)**.

1. Complete the table.

y = ½ F(x)

x	F(x)	y
–1	1	½
1	–1	– ½
2	–1	– ½
4	–2	–1



2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st one should be (–1, ½).*]
   These should be: (–1, ½), (1, – ½), (2, – ½), (4, – 1)
3. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   There was no change to the x-values.
4. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*
   Freddie Jr's y-values are one-half the y-values of Fred.
5. In y = ½ F(x), the coefficient of "F(x)" is ½. How did that affect the graph of Fred? How is this different from the graph of y = 4 F(x) on the previous page? The graph compressed vertically to half of its original size. It did not stretch horizontally. Instead of points getting 4 times farther from the x-axis, they got half as far from it.

VI. Checkpoint:

1. Complete each chart below. Each chart starts with the characteristic points of Fred.

x	F(x)	3 F(x)
–1	1	3
1	–1	–3
2	–1	–3
4	–2	–6

x	F(x)	¼ F(x)
–1	1	¼
1	–1	–¼
2	–1	–¼
4	–2	–½

2. Compare the 2nd and 3rd columns of each chart above. The 2nd column is the y-value for Fred. Can you make a conjecture about how a coefficient changes the parent graph?

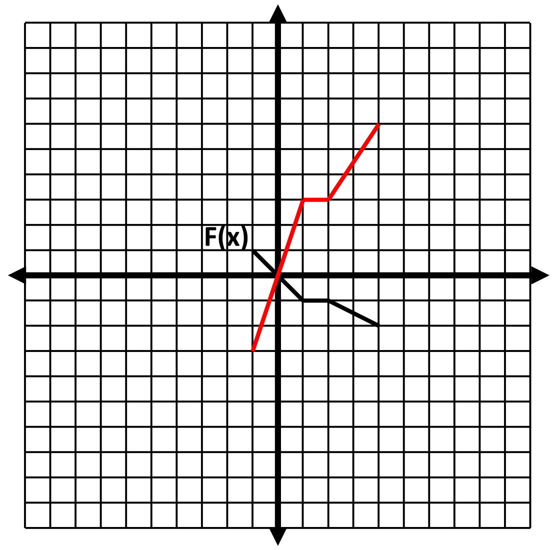
   Students will likely say that a coefficient greater than 1 stretches the graph (makes it taller/steeper) and a coefficient less than 1 compresses it (makes it shorter/less steep). This is not fully accurate but will be addressed in the next investigation.

VII. Now let's suppose that Freddie Jr. is **y = –3 F(x)**.

1. Complete the table.

y = –3 F(x)

x	F(x)	y
–1	1	–3
1	–1	3
2	–1	3
4	–2	6



2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st one should be (–1, –3).*]
   These should be: (–1, –3), (1, 3), (2, 3), (4, 6)
3. Reread the conjecture you made in #7 on the previous page. Does it hold true or do you need to refine it? If it does need some work, restate it more correctly here.
   If the *absolute value* of a coefficient is greater than 1, then it stretches the graph (makes it taller/steeper). If the *absolute value* of a coefficient is less than 1, then it compresses the graph (makes it shorter/less steep). A negative coefficient will reflect the graph in the x-axis.

VIII. Checkpoint: Let's revisit Harry, H(x).

1. Describe the effect on Harry's graph for each of the following.

   Example: –5H(x) Each point is reflected in the x-axis and is 5 times as far from the x-axis.

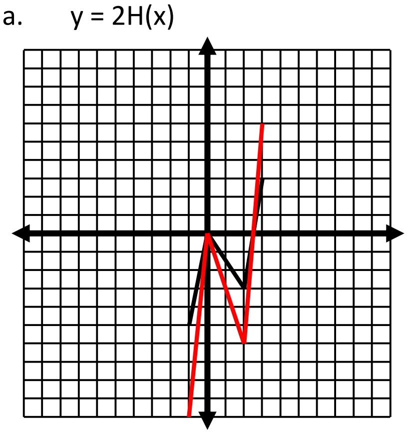
   a. 2H(x) Each point is twice as far from the x-axis.

   b. –2H(x) Each point is reflected in the x-axis and is twice as far from the x-axis.

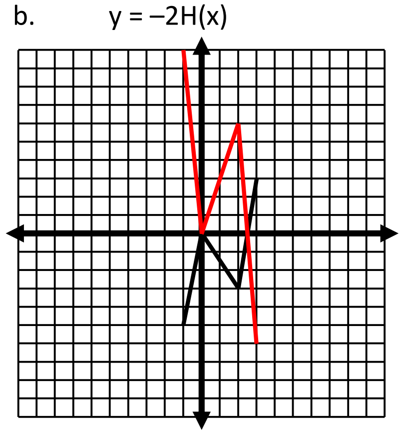
   c. $\frac{1}{2}$H(x) Each point is half as far from the x-axis.

2. Use your answers to questions 1 and 2 to help you sketch each graph *without using a table*.

a. y = 2H(x)



b. y = –2H(x)



c. y = $\frac{1}{2}$H(x)